Standard Cosmology

Homogeneous and isotropic universe

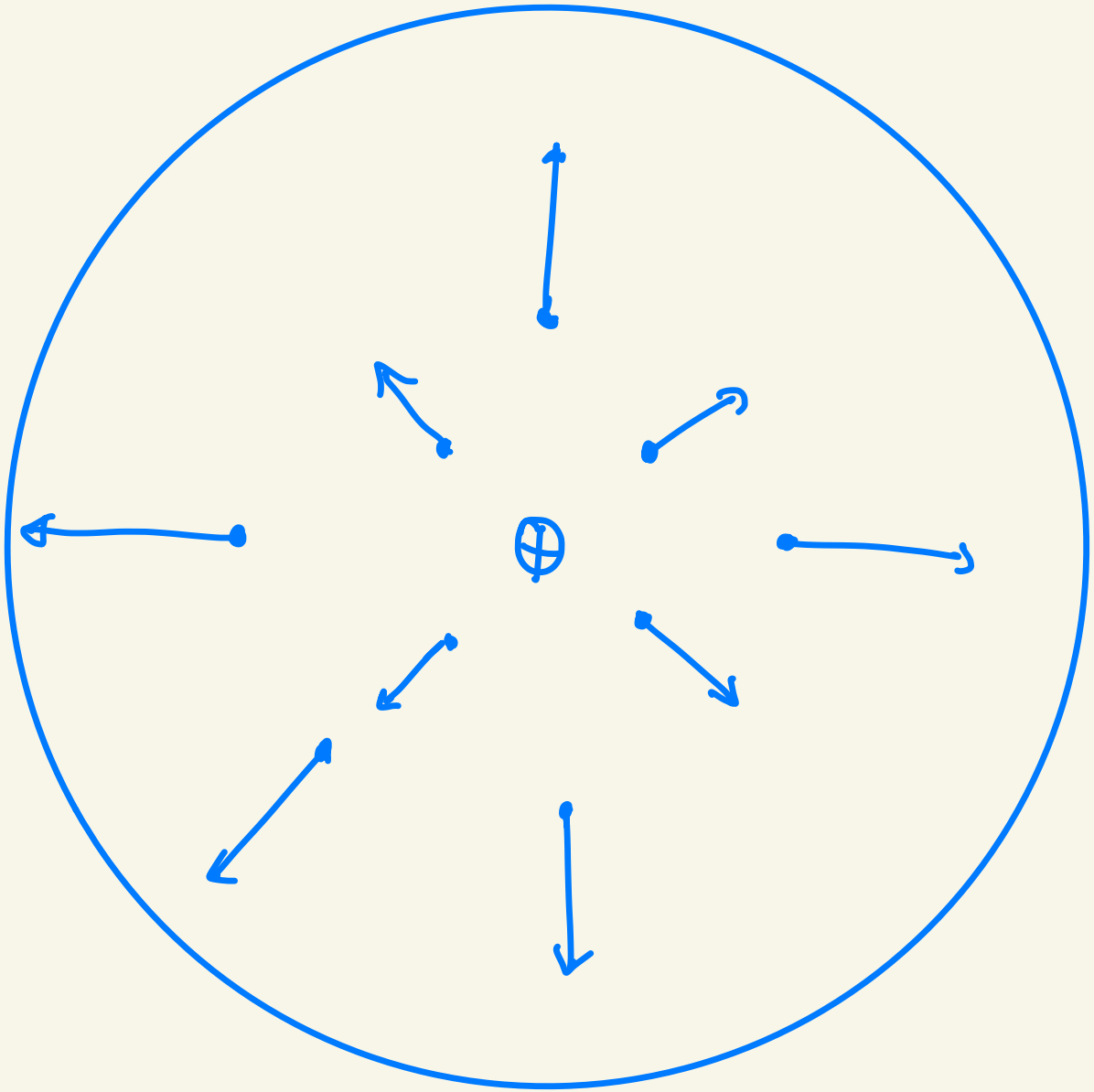
$$d\tau^2 = dt^2 - a^2\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

Friedmann - Lemaître - Robertson - Walker metric

k	1	0	-1
	closed	flat	open universe

$a(t)$ Scale factor

Hubble expansion



distance of free falling (comoving) objects ($k=0$)

$$d(t) = a(t)\, r$$

Coordinate distance, fixed

redshift $$z \equiv \frac{\lambda_0 - \lambda_i}{\lambda_i} = \frac{a_0}{a_i} - 1$$

Friedmann equations (from GR)

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3} G (\rho + 3P)$$

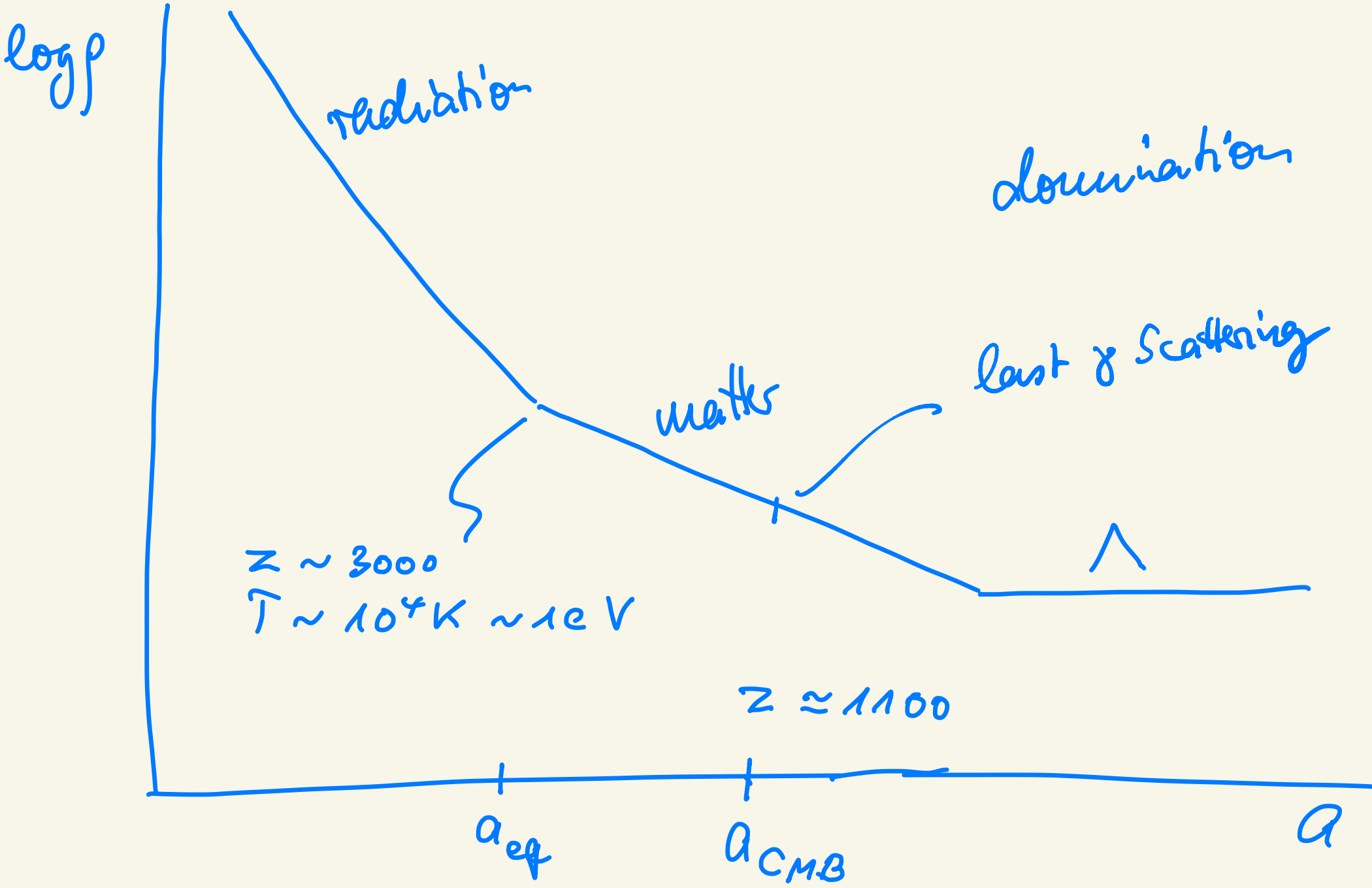
- G: Newton's const.
- ρ: energy density
- P: pressure

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3} G\rho$$

$$H \equiv \frac{\dot{a}}{a}$$ Hubble rate

$H_0 = 100\, h \text{ km s}^{-1} \text{Mpc}^{-1}$, $h = 0.674$

dominantly	ρ	P	$a(t)$
matter	a^{-3}	0	$t^{2/3}$
radiation	a^{-4}	$\frac{\rho}{3}$	$t^{1/2}$
Cosmological const.	Λ	$-\Lambda$	$\exp\left(t\sqrt{\Lambda/3}\right)$


log ρ
radiation
matter
Domination
last γ Scattering
z ~ 3000
T ~ 10⁴K ~ 1eV
z ≃ 1100
a_eq
a_CMB
a

Thermal equilibrium requires

$G = \frac{1}{m_{pl}^2}$

$t_{coll} \ll H^{-1}$ Hubble time

$t_{coll} \sim \frac{1}{\sigma n v}$, $H \sim \frac{\sqrt{\rho}}{m_{pl}}$

$m_{pl} \sim 10^{19}\,GeV$

high T: neglect masses

$\sigma n v \sim \alpha^2 T \sim 10^{-4} T$, $H \sim \frac{T^2}{m_{pl}}$

equilibrium for $T \ll 10^{-4} m_{pl} \sim 10^{15}\,GeV$

weak interactions for $T \ll m_W$:

$$\sigma v n \sim 10^{-2} \frac{T^5}{m_W^4}$$

equilibrium for $10^2 \frac{m_W^4}{T^5} \ll \frac{m_{pl}}{T^2} \Leftrightarrow$

$$T \gg \left(10^2 m_W^4 / m_{pl}\right)^{1/3} \sim 1\,\text{MeV}$$

neutrinos fall out of equilibrium at $T \sim 1\,\text{MeV}$

before e^+e^- annihilation $\Rightarrow$ $f = \frac{1}{e^{p/T}+1}$

$$T_\nu = \left(\frac{4}{11}\right)^{1/3} T_\gamma = 1.9\,\text{K} \sim \text{meV}$$

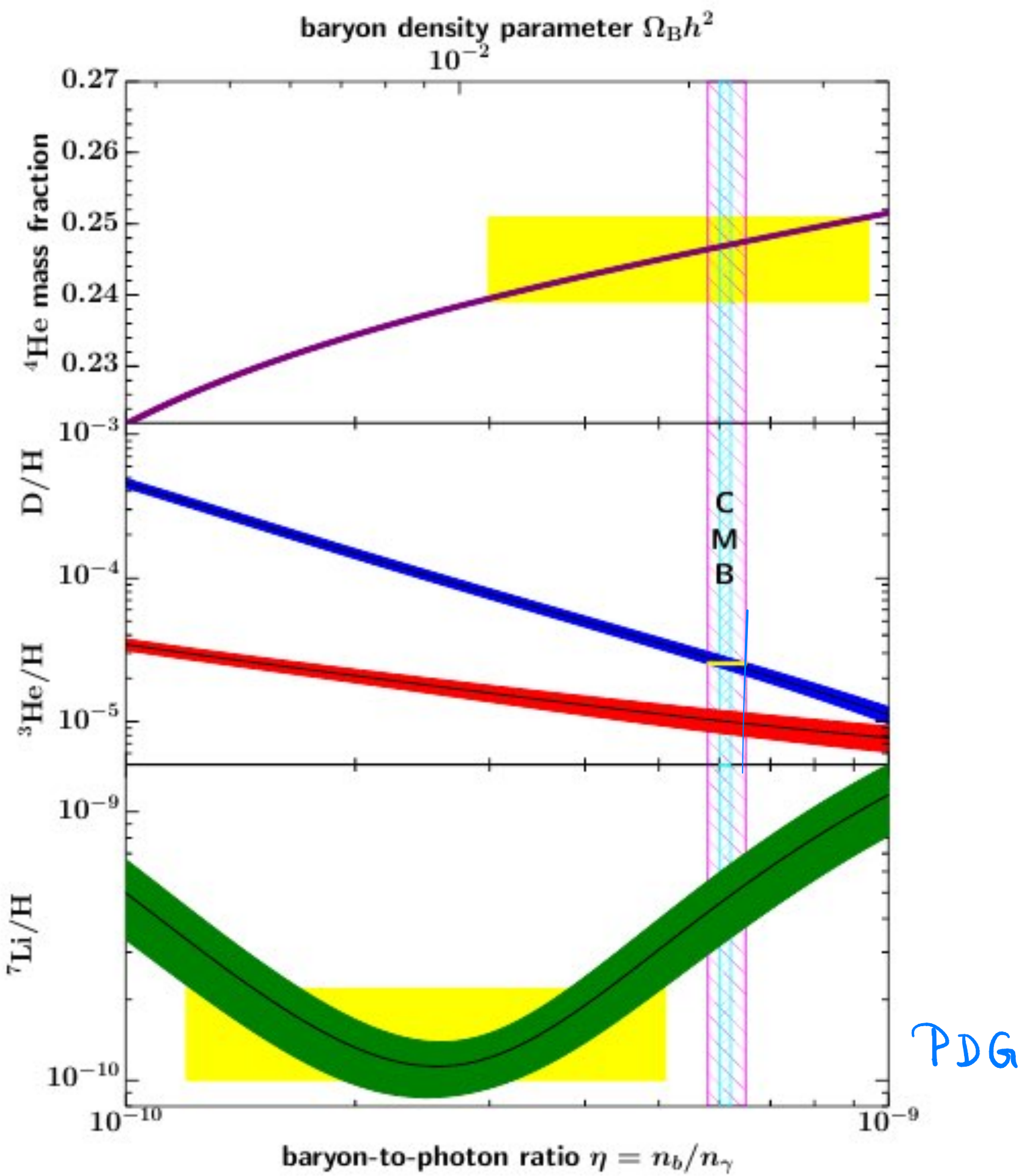
$100\,\text{MeV} \hat{=} 10^8\,\text{eV} \sim 10^{12}\,\text{K}$ $\quad 10^{-2}$ NR $\quad 10^{-5}$

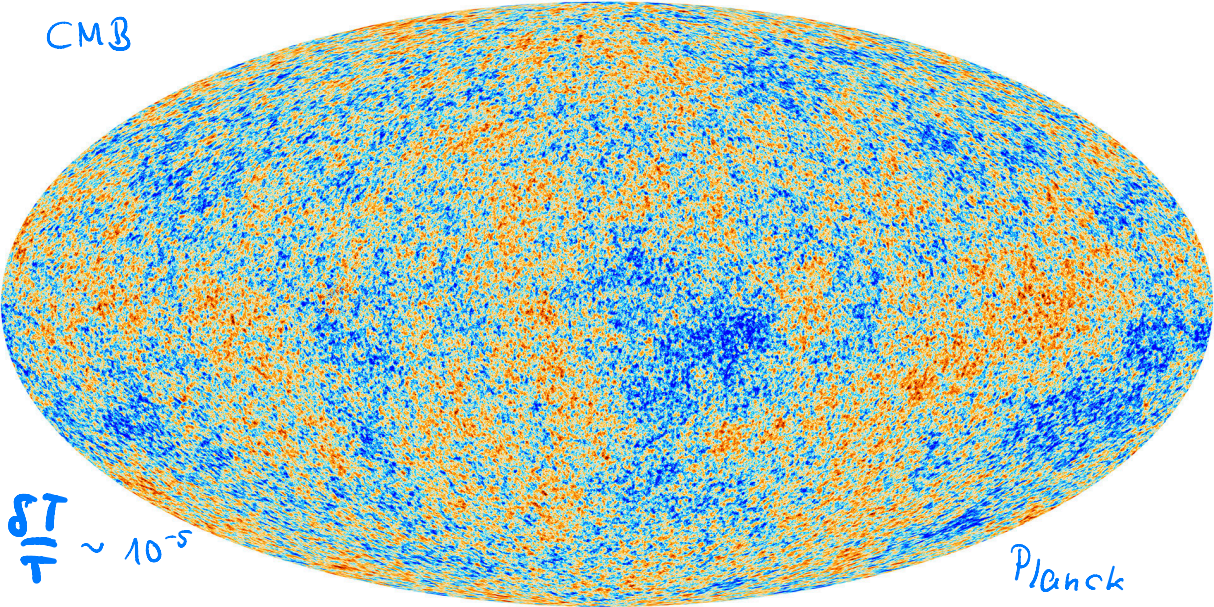
also at $T \sim 1\,\mathrm{MeV}$ $(t \sim 1\,\mathrm{s})$:

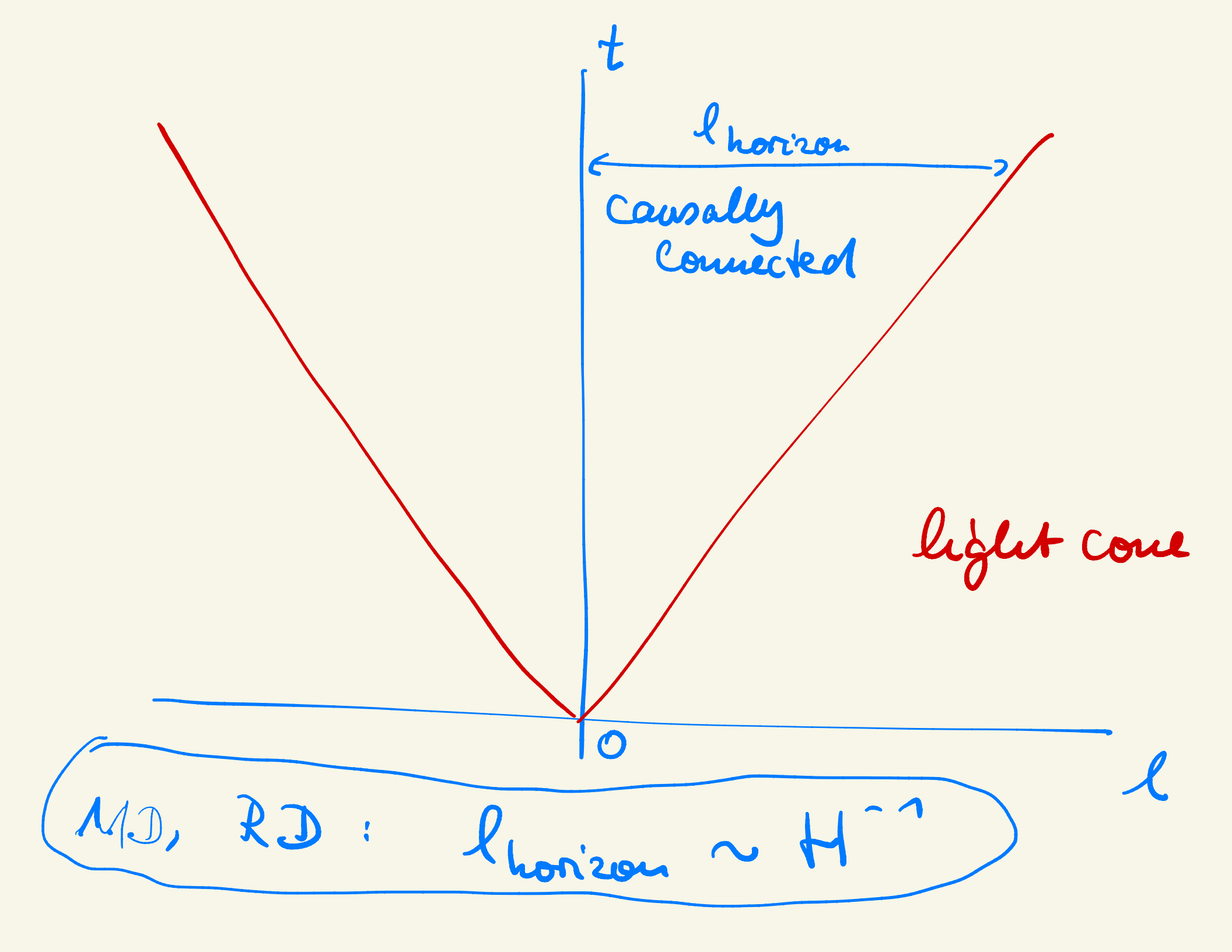
$\frac{n_p}{n_n}$ freezes $\longrightarrow$ initial cond.

for Big Bang Nucleosynthesis (BBN)

$$1\,\mathrm{s} \lesssim t \lesssim 3\,\mathrm{min}$$


baryon density parameter $\Omega_B h^2$
10^{-2}
0.27
0.26
0.25
0.24
0.23
^{4}He mass fraction
10^{-3}
D/H
10^{-4}
^{3}He/H
10^{-5}
C
M
B
10^{-9}
^{7}Li/H
10^{-10}
10^{-10}
10^{-9}
baryon-to-photon ratio $\eta = n_b/n_\gamma$
PDG

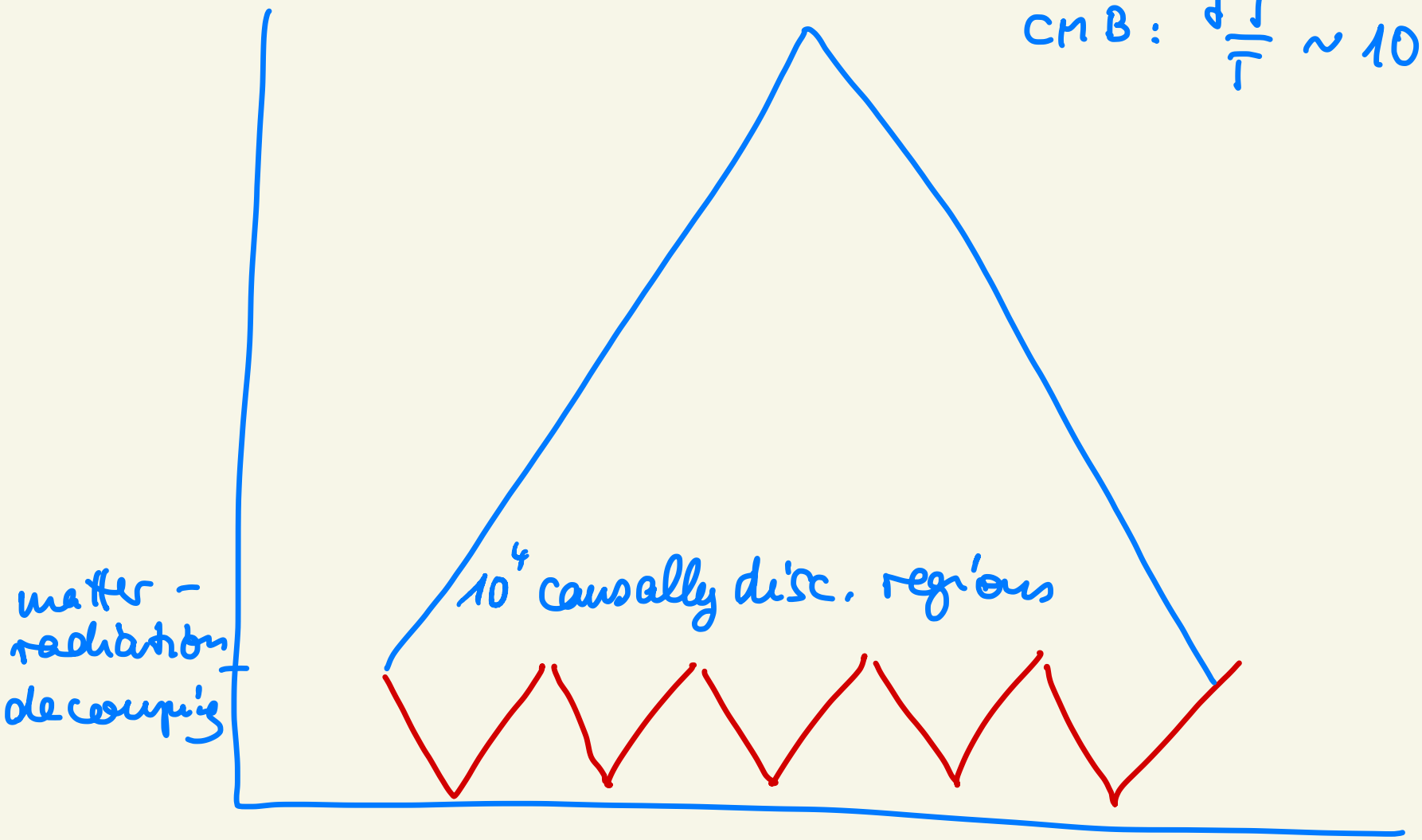

CMB
$\frac{\delta T}{T} \sim 10^{-5}$
Planck



MD, RD : $\ell_{horizon} \sim H^{-1}$

horizon problem

CMB: $\frac{\delta T}{T} \sim 10^{-5}$



density fluctuations $\rho = \bar{\rho} + \delta\rho$

short wavelengths : sound waves

$H^{-1} > \lambda \gtrsim \lambda_{Jeans}$ gravitational instability

$$\lambda_{Jeans}^2 \sim \frac{c_s^2}{G\bar{\rho}} = \frac{c_s^2 m_{pl}^2}{\bar{\rho}} \qquad (c_s : \text{sound speed})$$

Jeans length

→ structure formation

$$\ddot{\delta\rho} + (c_s^2 \vec{k}^2 - 4\pi G\bar{\rho})\,\delta\rho = 0$$

baryonic matter cannot form structure before photon decoupling

too late:

we need dark matter (DM)

DM perturbations can grow earlier

ΛCDM: 84% DM, 16% baryons

matter - antimatter asymmetry

$$\frac{n_B}{n_\gamma} = 6 \cdot 10^{-10}$$

$$n_B = n_b - n_{\bar{b}}$$

where does that come from ?

initial condition ?

inflation $\rightarrow$ $n_B = 0$

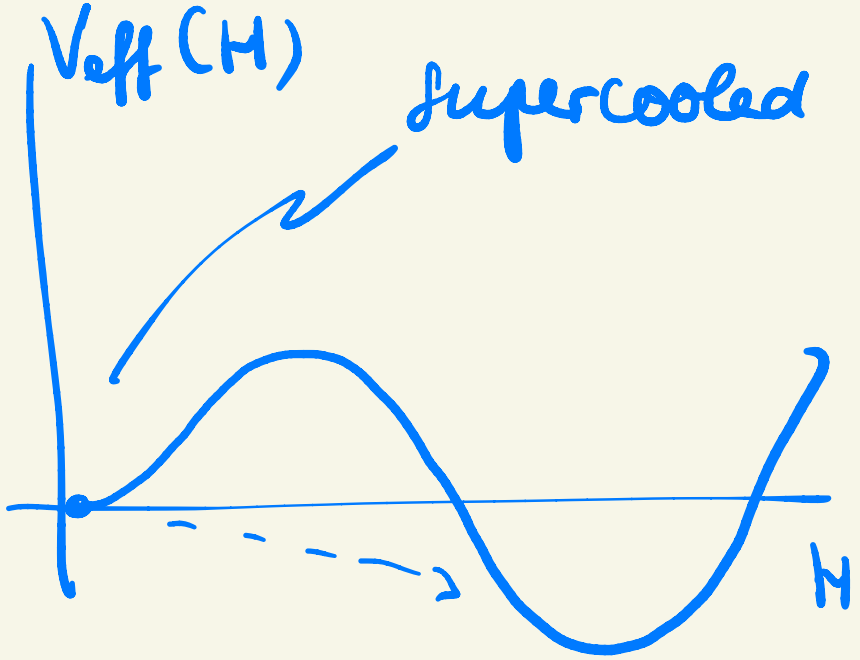
baryogenesis needs

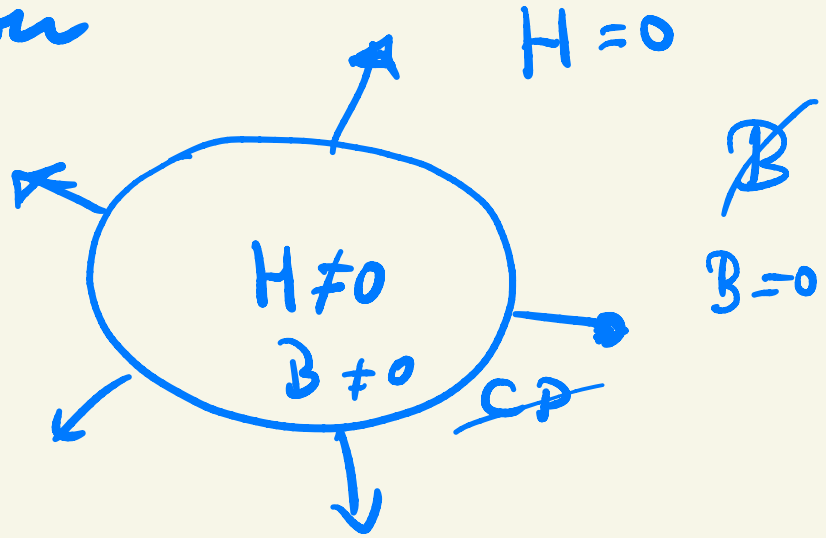
- non-equilibrium
- baryon number violation
- C- and CP- violation

all 3 cond's require physics beyond the SM

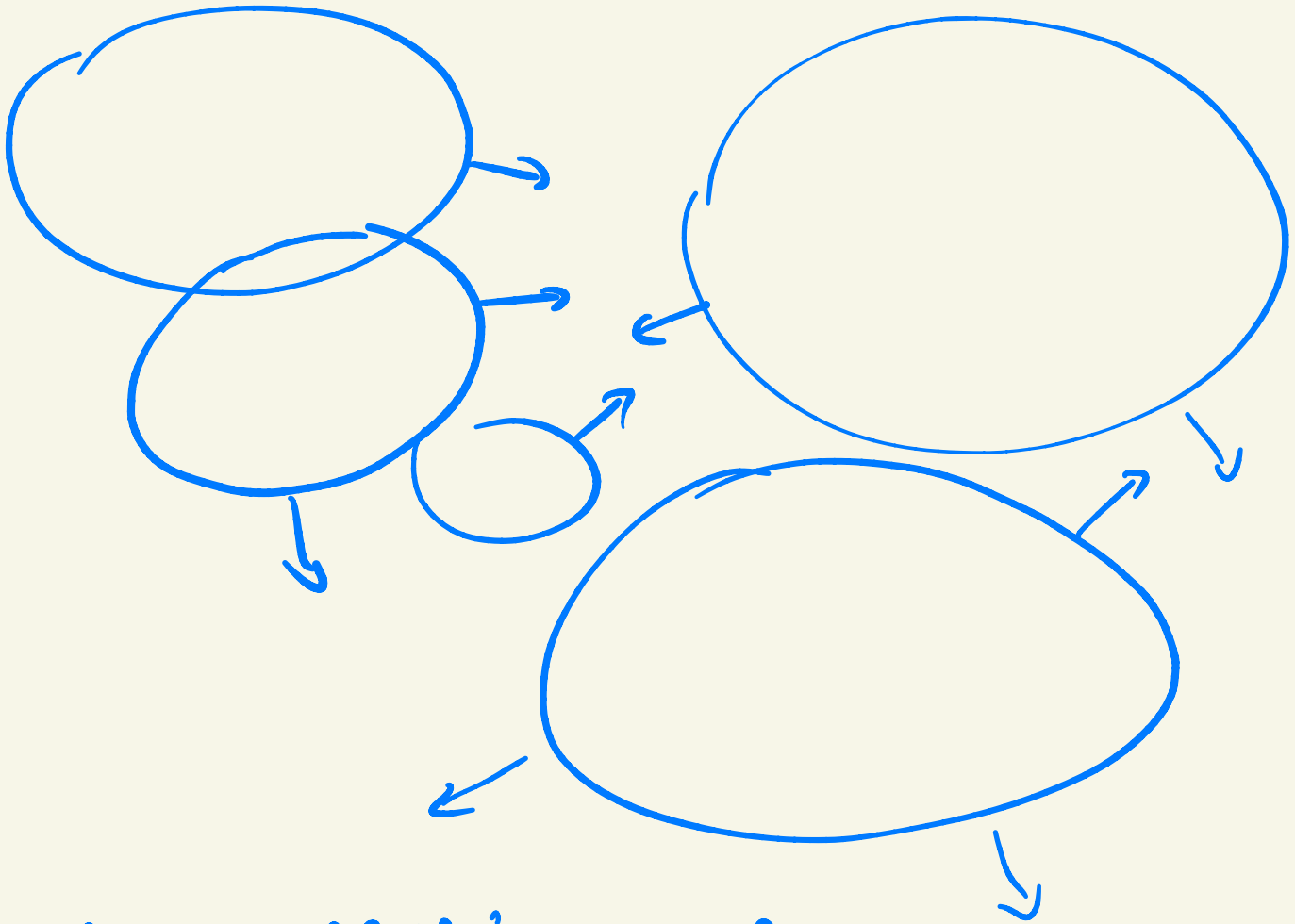
electroweak baryogenesis

first order phase transition





gravitational waves from 1st order phase transition

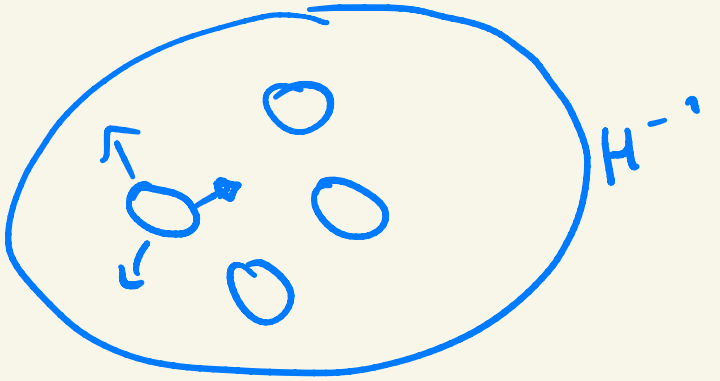


bubble collisions, sound waves → $Q_{ij} \neq 0$

electroweak phase transition : $T \sim 100\,\mathrm{GeV}$

$H_{EW} \sim \frac{10^4}{10^{19}}\,\mathrm{GeV} = 10^{-15}\,\mathrm{GeV} \sim 10^{-14}\,\mathrm{fm}^{-1} \sim 1\,\mathrm{m}^{-1}$

$\lambda_{EW} \lesssim H_{EW}^{-1}$

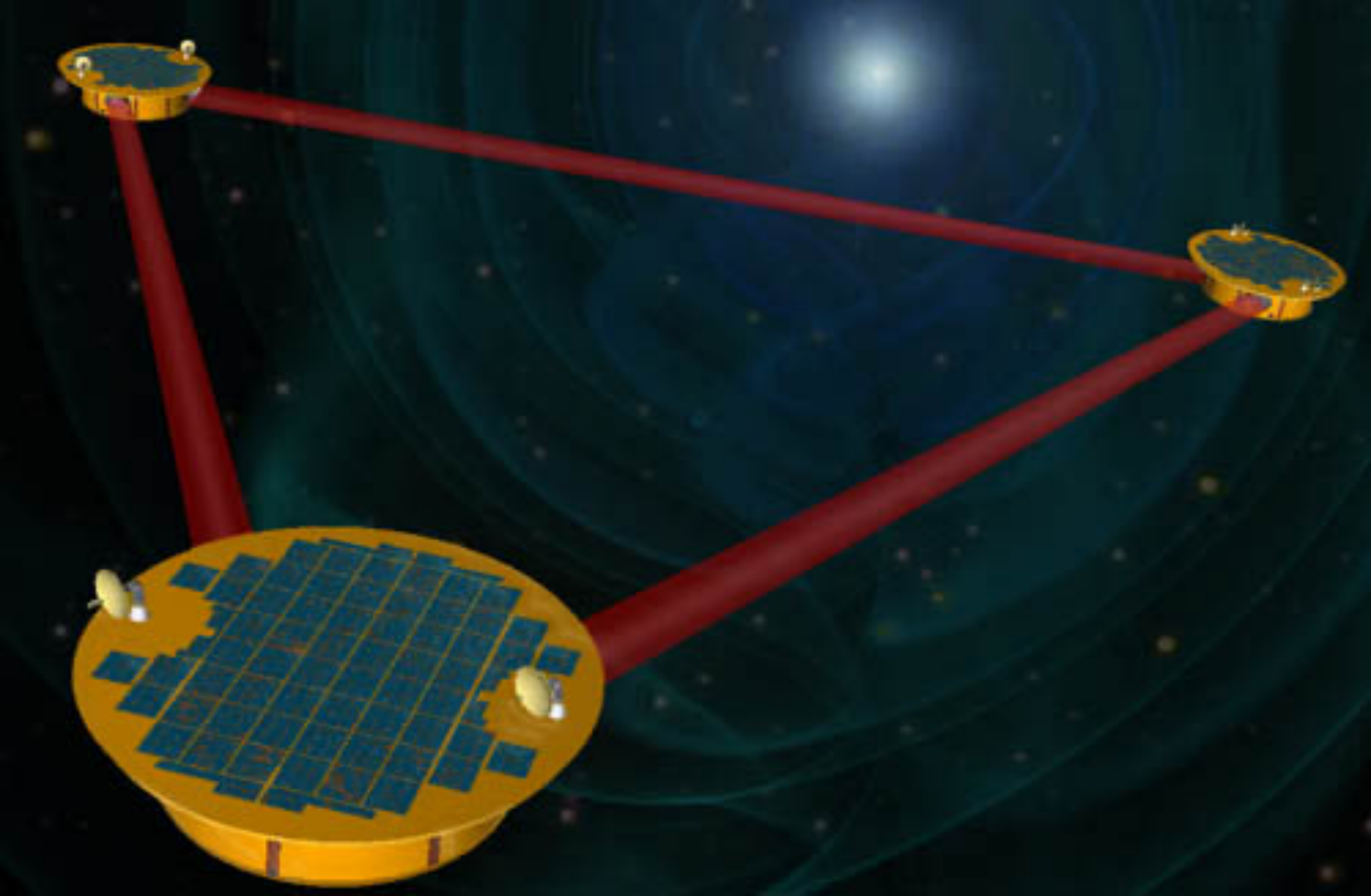


red shift: $z_{EW} \sim 10^{15} \Rightarrow \lambda_0 < 10^{15}\,\mathrm{m}$

$\nu_0 > 3 \cdot 10^8\,\mathrm{m/s} \cdot 10^{-15}\,\mathrm{m}^{-1} \sim 3 \cdot 10^{-7}\,\mathrm{Hz}$

LISA: $l \sim 1.5 \cdot 10^{11}\,\mathrm{m}$

(Laser interferometer Space antenna)



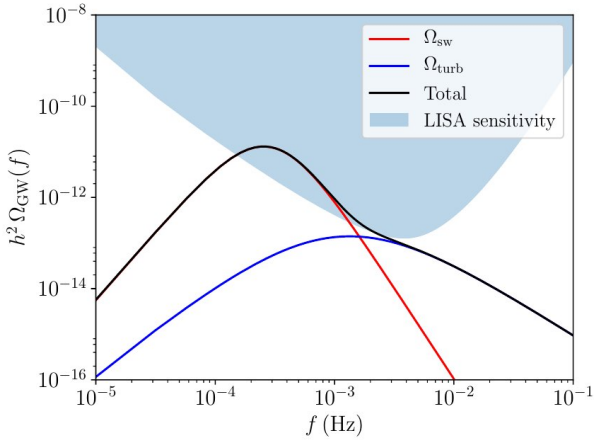


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Figure 6: Example of the gravitational wave power spectrum for a thermal phase transition, using the ansätze given in the text and with $v_{\rm w} = 0.44$, $\alpha_{T_*} = 0.084$, $H_*/\beta = 0.1$ and $T_* = 180\,\text{GeV}$ (see Section 6). The power spectrum is compared to a sensitivity curve obtained for a LISA-like configuration.

Summary

- early universe was nearly homogeneous & isotropic
- thermal equilibrium for $T \gtrsim 10^{15}$ GeV until CMB-decoupling ('recombination')
- oldest relics: CMB, BBN, $C\nu B$ (indirect)

future: $C\nu B$ (direct), gravitational waves from 1st order PT